NONLINEAR PROPERTIES DUST-ACOUSTIC ROGUE WAVES ON PLASMA SYSTEM

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ABSTRACT

The nonlinear propagation of dust-acoustic (DA) waves (DARWs) rogue waves has been investigated theoretically in a four components dusty plasma system containing negatively charged dust grains, isothermal ions, electrons and positrons following non-extensive q-distribution . The non-linear Schr¨odinger equation (NLSE) is derived by using a reductive perturbation method for this investigation. It has been observed that Modulation instability (MI) is significantly modified by the related plasma parameters (viz. dust masses, dust charge state, and non-extensive parameter q) . The findings of our present investigation may be useful for understanding different nonlinear electrostatic phenomena in both space (viz. ionosphere and mesosphere) and laboratory plasmas.

INTRODUCTION

The field of electron-positron-ion-dust (e-p-i-d) plasma physics is rapidly growing interest due to its existence in solar system, cosmic plasmas as well as laboratory plasmas. Most of the astrophysical plasmas usually contain highly charged (negative/positive) dust grains in addition to the electrons, positrons and ions because of the long time existence of positrons.

The space observations and laboratory experiments indicate the presence of particles which obey non-Maxwellian velocity distributions in plasmas. One of such distributions as reported by [16] afterwards proposed by [17] is the Boltzmann-Gibbs-Shannon(BGS) entropy in which the degree in which the degree of nonextensivity f the plasma particles.

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The propagation of wave packets in a nonlinear, dispersive medium is subject to the modulation of their wave amplitudes, i.e., a slow variation of the wave packets envelope due to the nonlinear self-interaction of the carrier wave modes.

The signature of the modulational instability (MI) and associated DARWs is observed and experimentally verified in various nonlinear sciences as well as the total picture of the MI and associated DARWs is organized by the nonlinear Schr¨odinger equation (NLSE).

A number of authors have studied various nonlinear waves in the plasma medium by considering q-distributed inertialess plasma species. Tasnim et-al [18] investigated the propagation of DA shock waves (DASHWs) in the q-distributed ionic plasma medium, and the magnitude of the amplitude of DASHWs decreases with non-extensive parameter-q.

Ferdousi et al [19] studied DASHWs in the presence of q-distributions and found that the polarity and amplitude of DASHWs depend on non-extensivity of ions. Saha and Chatterjee reported the generation and propogation of DA solitary waves (DASHWs) in a two component DP with q-distributed ions. Amour nd Tribeche [20] examined the DASWs in a DP with q-distributed electrons and found that non-extensivity of the electrons forms the DASW structure more spiky.

Emamuddin et al[21] investigated DAWs in a DP with ions and q-distributed electrons and observed that the amplitude of both positive negative Gardner solitons increases with non-extensivity. Ghosh et al studied the effect of the non-extensivity of ions during the head-ion collisin of DASWs and the phase shift in DP composed of dustand q-distributed ions.

Therefore,in our present work, we will derive an NLS equation by using a reductive perturbation method and study the nonlinear features of the DARWs in unmagnetized dusty plasmas comprising inertial warm negative dust, as well as inertialess isothermal ions and q-distributed electrons and positrons.

The manuscript is organized as follows: The basic governing equations of our considered plasma model is presented in Sec. II. The modulational instability is presented in Sec.IV. Rogue wave is included in Sec. IV the conclusion is provided in Sec. V.

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GOVERNING EQUATIONS

We consider an unmagnetized plasma system comprising of inertial warm negatively charged massive dust grains (mass md; charge qd = Zde), inertialess non-extensive distributed (electrons (mass me; charge e) and non-extensive distributed positrons (mass mp; charge +e) as well ions (mass mi; charge qi = +Ze), where Zd (Zi) is the number of electron (proton) residing on a negatively (positively) charged massive dust grains (ions). At equilibrium, the quasi-neutrality condition for our plasma model can be expressed as ne0 + Zdnd0 = np0 + Zini0.

where nd0 , ne0, np0, and ni0 are the equilibrium number densities of warm negatively charged dust grains, non- extensive (electrons and positrons) and ions respectively.

The normalized governing equations to study the DAWs are

$$\partial_t n_d + \partial_x (n_d u_d) = 0, \tag{1}$$

$$\partial_t u_d + u_d \partial_x u_d + e_1 n_d \partial_x n_d = \partial_x \phi, \tag{2}$$

$$\partial_{xx} \phi = e_2 n_e - e_3 n_p - (1 + e_2 - e_3)\, n_i + n_d \tag{3}$$

where n_d is the adiabatic dust grains number density normalized by its equilibrium value n_{d0}; u_d is the dust fluid speed normalized by the DA wave speed $C_d = (Z_d k_B T_i/m_d)^{1/2}$ (with T_i being the ion temperature, m_d being the dust grain mass, and k_B being the Boltzmann constant); ϕ is the electrostatic wave potential normalized by $k_B T_i/e$ (with e being the magnitude of single electron charge); the time and space variables are normalized by $\omega^{-1}_{pd} = (m_d/4\pi Z^2_d e^2 n_{d0})^{1/2}$ and $\lambda_{Dd} = (k_B T_i/4\pi Z_d n_{d0} e^2)^{1/2}$, respectively; $P_d = P_{d0}(N_d/n_{d0})^{\gamma}$ [with P_{d0} being the equilibrium adiabatic pressure of the dust, and $\gamma = (N + 2)/N$, where N is the degree of freedom and for one-dimensional case, $N = 1$ then $\gamma = 3$]; $P_{d0} = n_{d0} k_B T_d$ (with T_d being the temperatures of the adiabatic dust grains); and other plasma parameters are considered as $e_3 = n_{p0}/Z_d n_{d0}$, $e_2 = n_{e0}/Z_d n_{d0}$, $e_1 = 3T_d/Z_d T_i$, $e_4 = T_i/T_e$, $e_5 = T_i/T_p$,

Now, the expression fr the number densities of electrons and positrons following q-distributions are

$$n_e = \left[1 + (q-1) e_4 \phi\right]^{\frac{1+q}{2(q-1)}} \tag{4}$$

$$n_p = \left[1 - (q-1) e_5 \phi\right]^{\frac{1+q}{2(q-1)}} \tag{5}$$

Now, the expression for the number density of iso-thermal ions following the Maxwellian distribution can be written as

$$n_i = \exp(-\phi). \tag{6}$$

Now, by substituting Eqs. (4)-(6) in Eq. (3), and expanding up to third order of ϕ, we get

$$\partial_{xx}\phi + 1 = n_d + R_1\phi + R_2\phi^2 + R_3\phi^3 + \ldots \tag{7}$$

where

$$R_1 = [(q+1)(e_2e_4 + e_3e_5) + 2(1 + e_2 - e_3)]/2.$$
$$R_2 = [(q+1)(q-3)(e_3e_5^2 - e_2e_4^2) - 4(1 + e_2 - e_3)]/8.$$
$$R_3 = [(q+1)(q-3)(3q-5)(e_3e_5^3 + e_2e_4^3) + 8(1 + e_2 - e_3)]/48.$$

It may be noted here that the terms containing R_1, R_2, and R_3 in the right hand side of Eq. (7) are the contribution of inertialess electrons, positrons, and ions.

III. DERIVATION OF THE NLSE

To study the MI of the DAWs, we want to derive the NLSE by employing the reductive perturbation method, and for that case, first we can write the stretched co-ordinates in the form

$$\xi = \epsilon(x - v_g t), \tag{8}$$
$$\tau = \epsilon^2 t, \tag{9}$$

where v_g is the group speed and ϵ is a small parameter. Then we can write the dependent variables as

$$n_d = 1 + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} n_{dl}^{(m)}(\xi, \tau)\exp[il(kx - \omega t)], \tag{10}$$
$$u_d = \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} u_{dl}^{(m)}(\xi, \tau)\exp[il(kx - \omega t)], \tag{11}$$
$$\phi = \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\xi, \tau)\exp[il(kx - \omega t)], \tag{12}$$

where k and ω is real variables representing the carrier wave number and frequency, respectively. The derivative operators can be written as

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \in v_g \frac{\partial}{\partial \xi} + \in^2 \frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \in \frac{\partial}{\partial \xi}$$

Now, by substituting Eqs. (8)-(14) into Eqs. (1)- (2), and (7), and collecting the terms containing ϵ, the first order ($m = 1$ with $l = 1$) reduced equations can be written as

$$n_{d1}^{(1)} = \frac{k^2}{R}\phi_1^{(1)} , \tag{13}$$

$$u_{d1}^{(1)} = \frac{\kappa\omega}{R}\phi_1^{(1)}, \tag{14}$$

$$n_{d1}^{(1)} = -k^2\phi_1^{(1)} - R_1\phi_1^{(1)}, \tag{15}$$

where $R = e_1k^2 - \omega^2$. Hence these relation provides the dispersion relation for DAWs

$$\omega^2 = e_1k^2 + \frac{k^2}{R_1 + k^2}. \tag{16}$$

The coefficients of ϵ for $m = 2$ and $l = 2$ provide the second order harmonic amplitudes which are found to be proportional to $|\varphi_1^{(1)}|^2$

$$n_{d2}^{(2)} = R_4|\phi_1^{(1)}|^2, \tag{17}$$

$$u_{d2}^{(2)} = R_5|\phi_1^{(1)}|^2, \tag{18}$$

$$\phi_2^{(2)} = R_6|\phi_1^{(1)}|^2, \tag{19}$$

where

$$R_4 = \frac{2R_6k^2S^2 - e_1k^6 - 3\omega^2k^4}{2R^3},$$

$$R_5 = \frac{R_4\omega R^2\text{-}\omega k^4}{kR^2},$$

$$R_6 = \frac{e_1k^6 + 3\omega_2^2k^4 \quad 2R \;\; R^3}{6k^2R^3}.$$

Now, we consider the expression for (m = 3 with l = 0) and (m = 2 with l = 0), which leads the zeroth harmonic modes. Thus, we obtain

$$n_{do}^{(2)} = R_7\left|\phi_1^{(1)}\right|^2$$

$$u_{do}^{(2)} = R_8\left|\phi_1^{(1)}\right|^2$$

$$\phi_{do}^{(2)} = R_9\left|\phi_1^{(1)}\right|^2 \tag{20}$$

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where

$$R_7 = \frac{2\omega v_g k^3 + e_1 k^4 + \omega^2 k^2}{R^2(v_g^2 - e^1)} - \frac{R_9}{v_g^2 - e^{1}}$$

$$R_8 = \frac{R_7 v_g R^2 - 2\omega k^3}{R^2},$$

$$R_9 = \frac{2\omega v_g k^3 + e_1 k^4 + \omega^2 k^2 - 2R_2 S^2(e_1 - v_g^2)}{R^2(1 - R_1 v_g^2 + e_1 R_1)}.$$

Finally, the third harmonic modes ($m = 3$) and ($l = 1$), with the help of (15)-(27), give a set of equations, which can be reduced to the following NLSE:

$$i\frac{\partial \phi}{\partial \tau} + P\frac{\partial^2 \phi}{\partial \xi^2} + Q|\phi|^2 \phi = 0 \qquad (21)$$

where $\Phi = \phi_1^{(1)}$ for simplicity. In Eq. (27), P is the dispersion coefficient which can be written as

$$P = \frac{(v_g k - \omega)(\omega^3 + F_1) - R^3}{2\omega k^2 R}.$$

where $F_1 = 3k^2 e_1 \omega - 3v_g k\omega^2 - v_g k^3 e_1$ and also in Eq. (27), Q is the nonlinear coefficient which can be written as

$$Q = \frac{2R_2 R^2 (R_9 + R_6) + 3R_2 R^2 - F_2}{2\omega k^2}$$

where $F_2 = (k^4 e_1 + \omega^2 k^2)(R_4 + R_7) + 2\omega k^3 (R_8 + R_5)$ The space and time evolution of the DAWs in EPIDPM are directly governed by the coefficients P and Q, and indirectly governed by different plasma parameters such as e_1, e_2, e_3,e_4, and e_5,. Thus, these plasma parameters significantly affect the stability conditions of the DAWs.

MODULATIONAL INSTABILITY AND ROGUE WAVES

The stability of DAWs in four component plasma medium is governed by the sign of P and Q of the standard Eq. (27) [21]. When P and Q have same sign (i.e., $P/Q > 0$), the evolution of the DAWs amplitude is modulationally unstable in presence of the external perturbations. On the other hand, when P and Q have opposite sign (i.e., $P/Q < 0$), the DAWs are modulationally stable in presence of the external perturbations. The plot of P/Q against k yields stable and unstable parametric regimes of the DAWs. The point, at which transition of P/Q curve intersect with k-axis,

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is known as threshold or critical wave number k (= k_c) The governing equation for the highly energetic DARWs in the unstable region ($P/Q > 0$) can be written as

$$\phi(\xi,\tau)=\sqrt{\frac{2P}{Q}}\left[\frac{4(1+4iP\tau)}{1+16P^2\tau^2+4\xi^2}-1\right]\exp(2iP\tau)$$

Equation (29) describes that a large amount of wave energy, which causes due to the nonlinear characteristics of the medium, is localized into a comparatively small area in space.

CONCLUSION

In this study, we have performed a nonlinear analysis of DAWs in a four component EPIDPM consisting of inertial negatively charged massive dust grains, and inertialess non-thermal Cairns' distributed electrons as well as iso-thermal positrons and ions. The evolution of the DAWs is governed by the standard NLSE and the coefficients P and Q of NLSE represent the stable/unstable parametric regimes of DAWs in the presence of the external perturbation. The present results could be applied for in-depth understanding of the nonlinear electrostatic structure in astrophysical environments especially in interstellar and spatial oservations where non-extensive (electrons and positron) distributions may be present.

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